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Multi fluidity and Solitary wave stability in cold quark matter: core of dense astrophysical objects

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Abstract. Considering the magneto-hydrodynamic equations in a non-relativistic multi fluid framework, we study the behavior of small amplitude perturbations in cold quark matter. Magneto-hydrodynamic equations, along with a suitable equation of state for the cold quark matter, are expanded using the reductive perturbation method. It is shown that in small amplitude approximation, such a medium should be considered as a multi-fluid system. The result is a nonlinear wave equation which complies with a modified form of the derivative nonlinear Schrodinger equation instead of the KdV equation. Considering the magnetic field which is supported by the Maxwell's equations, we show that the complete set of equations, create stable solitary waves. An interesting result is the existence of an electric field component along the direction of magnetic field which causes a small charge separability in the medium. Properties of this solitonic solution are studied by considering different values for the environmental characters such as background mass density and strength of the magnetic field (at the scale of compact stars).

1 Introduction

Soon after establishing the idea of asymptotic freedom in quantum chromodynamics (QCD), possibility of the existence of quark gluon plasma (QGP) or quark matter at high temperature (above $150 \sim 200MeV$) and/or high density (upper than the nuclear density, $\rho_0 \sim 3.0 \times 10^{17} Kg/m^3$) was established [1, 2, 3, 4]. Based on these two conditions, we should expect to find QGP in few milliseconds after the Big Bang [3, 5], at initial states of high energy heavy ion collisions, which is currently being experimentally pursued [3, 6], and in the core of super dense astronomical objects, such as neutron stars, magnetars and quark stars [2, 3, 7, 8].

At sufficiently high densities and low temperatures, as in the dense interior of massive neutron stars, hadrons are melted into cold quark matter consist of a Fermi sea of free quarks [9, 10]. Nowadays, the astronomical observations indicate the existence of huge magnetic field, from $10^8 T$ at the surface to $10^{13} T$ in the core of neutron stars [11, 12, 13]. Also, the collapse of a white dwarf to a neutron star happens with an extremely strong magnetic field [14]. Therefore, it is not surprising that a large number of recent works have been presented to investigate the role of strong magnetic fields on the behavior of dense quark matter.

But, there are few studies on the collective behavior and long range interactions in the quark matter and QGP, modified by magnetic fields in the framework of magneto-hydrodynamics (MHD). The set of magneto-hydrodynamic equations are a combination of: I) fluid dynamic equations which the Navier-Stokes equation is essentially the simplest equation describing the motion of a fluid in the non-relativistic framework [15], II) the continuity equations and III) the Maxwell's equations. These differential equations with different types of nonlinearity and dispersion terms can be solved mainly numerically. Important characters of propagating waves inside these media are derived by using an approximation method which is able to save the nonlinear behavior of equations and provide small amplitude solutions describing long range effects in the system. This is the Reductive Perturbation Method (RPM) which is a helpful technique preserving nonlinear, dispersive and dissipative effects of the QGP medium in relevant differential equations [16, 17].

A number of striking works on the propagation of nonlinear waves inside the QGP, mostly in the absence of magnetic field through the continuity and momentum equations and considering different models for the equation of state, have been presented which predict the existence of unstable long range behaviors in the framework of the KdV like equations with breaking and shock profiles [17, 18, 19]. Almost all presented equations of state, containing special parts which provide a nonlinear term in the equation of motion. But for establishing stable solitonic solution in the framework of KdV equation, we need a dispersion term too. This part of the KdV equation can be created through a Laplacian term in the energy density which adds a cubic derivative respect to the space coordinate. In the electromagnetic plasmas, this issue is provided by the Laplace equation of the electric potential while in the QGP, there is not same situation at the first order of approximation. Indeed, the most accepted field theoretical models do not have higher derivative terms in their leading orders. Such terms may be appeared in higher orders of approximations which are usually negligible [20]. One can expand the mass density according to the derivatives of space coordinate, if the mass density of the medium is not in its equilibrium state [21]. Therefore, higher derivatives are able to create the dispersive term in the KdV equation.

The gluon field in cold quark gluon plasmas is very large, because of high densities in these situations. If we assume that the coupling constant is not very small and is not spatially constant, the existence of intense gluon field implies that the bosonic fields tend to have large occupation numbers, and therefore higher derivative terms is appeared in the spatial expansion of related term in the energy density [22]. Using above models (as well as other methods), one can add a weak dispersive term to the KdV equation; however it is sufficient for stabilizing small amplitude solitary waves. We have shown that such localized effects even in the shape of breaking waves are long lasting enough to create detectable signatures in the border of QGP medium [23].

Most of the QGP media have been created in an electromagnetically rich environments. Neutron stars, pulsars and magnetars are examples of compact astrophysical objects where the cold and dense nuclear matter and/or QGP may exist. The magnetic field in such objects typically is very high. This means that investigating behavior of these media without considering the magnetic field can not be realistic. It is shown that presenting a fixed extreme but external magnetic field stabilizes breaking or shock waves in cold QGP media. In other words, if external magnetic field is enough strong, breaking waves are changed into stable solitary profiles which are governed by the ZakharovKuznetsov (ZK) equation [24] (but not a KdV equation!). In such situations, we have not need to consider small nonlinear terms which are generally so weak. Indeed, the solitonic solutions of the ZK model are stable by themselves. A realistic result can be extracted through solving the full equations of the system by considering the internal magnetic field created in the QGP matter by itself. According to our best of knowledge, evolution of localized waves due to collective behaviors and long rage interactions by considering the full equations of the system has not been investigated before. It is clear that without these results, our knowledge about the wave propagation in such media is incomplete. We will see the results are completely different with our previous knowledge. Here, we focus on models in which the quark matter or the QGP environment consist of two light flavors (u and d quarks) and a high flavor (s quark). The bulk QGP is given as a near ideal Fermi liquid and the mQCD model is used as equation of state. It may be noted that this model is not able to create any dispersive term. Cold QGP in an external magnetic field as a multi fluid system has been investigated recently, but without considering the self magnetic field due to QGP constituents [25]. Stability conditions have been investigated in this paper but propagation of nonlinear waves has not been studied. Indeed, we have to consider induced magnetic field due to motion of plasma particles as well as nonlinear wave propagation to find more realistic knowledge about the system and collective effects therein.

Outlines of this paper are as follows: In the next section we briefly present the magnetohydrodynamic equations in non-relativistic QGP. In the section III, a review for the QGP equation of state according to the mQCD model is given. We expand the system of equations using the reductive perturbation method and derive a nonlinear equation for system variables at zero temperature, in the section IV. The solitonic solution for the derived equation is obtained in section V. In the section VI we discuss on the properties of the localized wave describing the evolution of transverse magnetic field perturbations and the last section is devoted to some concluding remarks.

2 Magneto-hydrodynamic equations of QGP

The core of neutron stars which consist of cold QGP or quark matter is known as a superfluid. QGP species have different charges and due to the magnetic field, they find different trajectories in their media. Although, the whole fluid motion is supported by a certain velocity, every particle can take a specific velocity which is different and the effect of this difference in creating perturbation is important. This means that we have to apply the multi fluid approach to describe the system equation of motion [24, 25, 26]. Separation of QGP constituents in term of their charges due to strong magnetic field has also been proven experimentally [27, 28].

For describing the magneto-hydrodynamics, we can write a system of equations which are governed by a conservation law, the energy-momentum equations of motion and evolution of the electromagnetic field through the Maxwell's equations[17, 24, 29, 30]. For simplifying, we suppose that QGP consists of three flavors u, d, s which have the following masses: $m_u = 2.2MeV$, $m_d = 4.7MeV$ and $m_s = 96MeV$ and their respective charges are: $Q_u = 2Q_e/3$, $Q_d = -Q_e/3$ and $Q_s = -Q_e/3$ where Q_e is the absolute value of the electron charge.So, the set of equations can be written as following:

$$\frac{\partial \rho_{Bi}}{\partial t} + \nabla.(\rho_{Bi}\mathbf{v}_i) = 0 \tag{1}$$

$$\rho_{mi}\left(\frac{\partial}{\partial t} + \mathbf{v}_i.\nabla\right)\mathbf{v}_i = -\nabla p_i + \rho_{ci}(\mathbf{E} + \mathbf{v}_i \times \mathbf{B}) \tag{2}$$

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E} \tag{3}$$

$$\nabla \times \mathbf{B} - \epsilon\mu\frac{\partial \mathbf{E}}{\partial t} = \mu\sum_i \rho_{ci}\mathbf{v}_i \tag{4}$$

$$\nabla.\mathbf{B} = 0 \tag{5}$$

$$\nabla.\mathbf{E} = \frac{1}{\epsilon}\sum_i \rho_{ci} \tag{6}$$

Equations (1) and (2) are the baryon density continuity equation and non relativistic equation of motion, where ρ_{mi}, ρ_{Bi}, ρ_{c_i} and $\mathbf{v}_i$ denote, the mass density, baryon density, charge density and the velocity of each quark's favor respectively. Also, the equations (3)-(6) are the Maxwell's equations with the magnetic field $\mathbf{B}$ and the electric vector field $\mathbf{E}$, while ϵ and μ are effective dielectric constant and magnetic permeability respectively. If we assume that the net charge of quark mixture is negligible, the global charge neutrality are enforced by [29, 30, 31]:

$$\rho_{cu} = \rho_{cd} + \rho_{cs} \tag{7}$$

and the baryon number density (ρ_B) conservation implies that [24, 32]:

$$\rho_B = \frac{1}{3}(\rho_u + \rho_d + \rho_s) \tag{8}$$

where ρ_i is the quark number density.

Now, we are trying to combine three sets of momentum equations for three different particles into one set of equations. Because of the almost identical mass of light quarks (u, d) respect to the s quark, with a good approximation, we can assume that these two types of quarks have the same velocity ($\mathbf{v_u} \simeq \mathbf{v_d} = \mathbf{v}'$). So that by ignoring the displacement current[30] and using (7), the equation (4) becomes:

$$\nabla \times \mathbf{B} = \mu\rho_{cs}(\mathbf{v}' - \mathbf{v}) \tag{9}$$

where $\mathbf{v}$ is the velocity of s quark. By substituting velocity of light quarks from above expression into (2) and gathering u, d index, we obtain the electric field in terms of velocity of s quark and magnetic field. Replacing this electric field in equation of motion for s quark gives us the following equation:

$$\begin{aligned}\rho_{mQGP}\frac{d\mathbf{v}_s}{dt} = \quad & -\nabla p + \frac{1}{\mu}(\nabla \times \mathbf{B}) \times \mathbf{B} \\ & -\frac{\rho_{mu} + \rho_{md}}{\mu\rho_{cs}}\left[\{(\nabla \times \mathbf{B}).\nabla\}\mathbf{v} + \frac{d}{dt}(\nabla \times \mathbf{B})\right] \\ & -\frac{\rho_{mu} + \rho_{md}}{(\mu\rho_{cs})^2}\{(\nabla \times \mathbf{B}).\nabla\}(\nabla \times \mathbf{B})\end{aligned} \tag{10}$$

and supplanting the electric field by using of s quark's equation of motion (2) in (3) results:

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) - \frac{\rho_{ms}}{\rho_{cs}}\left(\nabla \times \frac{d\mathbf{v}}{dt}\right) \tag{11}$$

where d/dt denotes $\partial/\partial t + (\mathbf{v}.\nabla)$. In general, the relationship between the mass density ρ_m and the particle density (ρ) is given by $\rho_m = m\rho$, where m is the particle mass. So, in term of baryon number density we can write: $\rho_{mf} = 3m_f\rho_{Bf}$. For simplifying equations, we need an additional condition which is supposed as following:

$$\rho_u + \rho_d = \alpha\rho_s \tag{12}$$

by using this equation , (7) and (8), we can find:

$$\rho_s = \frac{3}{1+\alpha}\rho_B \tag{13}$$

and

$$\rho_{mu} + \rho_{md} = \rho_{mQGP} - \rho_{ms} = \frac{3\alpha}{1+\alpha}m_s\rho_B \tag{14}$$

so that the set of MHD equations becomes:

$$\frac{\partial \rho_B}{\partial t} + \nabla.(\rho_B \mathbf{v}) = 0 \tag{15}$$

$$\begin{aligned}\rho_{mQGP}\frac{d\mathbf{v}}{dt} = \quad & -\nabla p + \frac{1}{\mu}(\nabla \times \mathbf{B}) \times \mathbf{B} \\ & -\frac{\alpha m_s}{\mu|Q_s|}\left[\{(\nabla \times \mathbf{B}).\nabla\}\mathbf{v} + \frac{d}{dt}(\nabla \times \mathbf{B})\right] \\ & -\frac{\alpha(1+\alpha)m_s}{3(\mu|Q_s|)^2\rho_B}\{(\nabla \times \mathbf{B}).\nabla\}(\nabla \times \mathbf{B})\end{aligned} \tag{16}$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) + \frac{m_s}{|Q_s|}\left(\nabla \times \frac{d\mathbf{v}}{dt}\right) \tag{17}$$

In this step, we have derived a complete set of equations ((15, (16) and (17)) which describe time evolution of the cold QGP system. Unfortunately, above equations are highly nonlinear. We have not allowed to ignore dispersion, dissipation and nonlinear terms as they play essential roles in the dynamics of the system. Luckily, it is possible to investigate the behavior of the system by considering all above terms using the reductive perturbation method (RPM) for small amplitude excitations [16, 18, 33].

3 The QGP equation of state

The equation set (15)-(17) can not be evaluated only if we add another equation, describing the fluid pressure (p) respect to other variables of the system, which calls equation of state (EOS). Several equation of states have been proposed for QGP through different approaches. The MIT bag model [34], strongly interacting QGP model [35], Cornell potential model [36] are some of the most famous presented EOS for QGP. Another version of MIT bag model with negative bag constant has been presented [37]. This new EOS satisfies needed conditions provided in the lattice QCD simulations. The energy density in this model is also similar to that in the standard bag model, but with a negative value of the bag constant. Therefore, one can consider modified MIT bag model in calculations. It may be noted that there is not any model which is completely accepted by physics community. We have used the equation of state which is called mQCD and was derived in [31, 38]. This model comprises the effect of magnetic field in the EOS; however, similar procedure can be applied for other forms of EOS.

The energy density ε, the parallel pressure ($p_{||}$) and the perpendicular pressure ($p_\perp$) are given respectively as following[31, 38, 39]:

$$\varepsilon = \frac{27g_h^2}{16m_G^2}\rho_B^2 + \mathcal{B} + \frac{B^2}{8\pi} + \sum_{f=u}^{d,s}\frac{|Q_f|B}{2\pi^2}\sum_{n=0}^{n_{max}^f}3(2-\delta_{n,0})\int_0^{k_{z,F}^f(n)}dk_z\sqrt{k_z^2+m_f^2+2n|Q_f|B} \tag{18}$$

$$p_{||} = \frac{27g_h^2}{16m_G^2}\rho_B^2 - \mathcal{B} - \frac{B^2}{8\pi} + \sum_{f=u}^{d,s}\frac{|Q_f|B}{2\pi^2}\sum_{n=0}^{n_{max}^f}3(2-\delta_{n,0})\int_0^{k_{z,F}^f(n)}dk_z\frac{k_z^2}{\sqrt{k_z^2+m_f^2+2n|Q_f|B}} \tag{19}$$

$$p_{\perp} = \frac{27g_h^2}{16m_G^2}\rho_B^2 - \mathcal{B} + \frac{B^2}{8\pi} + \sum_{f=u}^{d,s} \frac{|Q_f|^2 B^2}{2\pi^2} \sum_{n=0}^{n_{max}^f} 3(2-\delta_{n,0})n \int_0^{k_{z,F}^f(n)} \frac{dk_z}{\sqrt{k_z^2 + m_f^2 + 2n|Q_f|B}} \tag{20}$$

The baryon density is given by:

$$\rho_B = \sum_{f=u}^{d,s} \frac{|Q_f|B}{2\pi^2} \sum_{n=0}^{n_{max}^f} (2-\delta_{n,0})\sqrt{\mu_f^2 - m_f^2 - 2n|Q_f|B} \tag{21}$$

with:

$$n \leq n_{max}^f = int\left[\frac{\mu_f^2 - m_f^2}{2|Q_f|B}\right] \tag{22}$$

where $int[a]$ explains the integer part of a and mu_f is the chemical potential for the quark f. It is defined in [31]$\xi = \frac{g_h}{m_G}$ and the MIT bag model that EOS is recovered by selecting $\xi = 0$. For a given magnetic field intensity, we choose the values for the chemical potentials mu_f which determine the density ρ_B. We also choose the other parameters such as ξ and $\mathcal{B}$. In this case, the pressure gradient which we should place in magneto-hydrodynamic equations, becomes:

$$\nabla p = \frac{27g_h^2}{8m_G^2}\left(\rho_B\frac{\partial\rho_B}{\partial x}, \rho_B\frac{\partial\rho_B}{\partial y}, \rho_B\frac{\partial\rho_B}{\partial z}\right) \tag{23}$$

Also, in the non relativistic limit $\varepsilon \cong \rho_m$ [17, 19] can be used in MHD equations.

In the next section, we present time evolution equations of the system by introducing suitable variables using the RPM formalism.

4 The reductive perturbation method

This method is a very powerful way of deriving simplified models describing nonlinear wave propagation and is based on perturbation expansion which all variables are expanded around their equilibrium values in terms of the small expansion parameter ϵ [18, 19, 24]. Then, we apply a stretched coordinates which are connected to Cartesian coordinates. Substituting the new coordinates and expansion of variables give us the differential equation(s) which govern the space time evolution of the perturbation. Consider infinitely extended uniform cold QGP which its physical quantities at equilibrium state are given by [40]:

$$\rho_B = \rho_{B0} \qquad \mathbf{B} = B_0\hat{e}_x \qquad \mathbf{v} = 0$$

Components of velocity and magnetic field vectors are defined as: $\mathbf{v} = (v_x, v_y, v_z)$ and $\mathbf{B} = (B_x, B_y, B_z)$. Transverse components of velocity ($\tilde{v}$) and magnetic field ($\tilde{B}$) can be introduced by following complex quantities:

$$\tilde{v} = v_y + iv_z \qquad \tilde{B} = B_y + iB_z \tag{24}$$

We define the stretched coordinates as:

$$\begin{aligned} \xi &= \epsilon(x - \lambda t) & \eta &= \epsilon^{3/2}y \\ \zeta &= \epsilon^{3/2}z & \tau &= \epsilon^2 t \end{aligned} \tag{25}$$

where λ is the wave velocity at linear approximation and ϵ is the expansion parameter, i.e. $\epsilon < 1$. Here, we can expand the variables in power series of ϵ as follows:

$$\begin{aligned}
&\rho_B = \rho_{B0} + \epsilon\rho_B^{(1)} + ...\\
&\tilde{B} = \epsilon^{1/2}(\tilde{B}^{(1)} + \epsilon\tilde{B}^{(2)} + ...)\\
&\tilde{v} = \epsilon^{1/2}(\tilde{v}^{(1)} + \epsilon\tilde{v}^{(2)} + ...)\\
&B_x = B_0 + \epsilon B_x^{(1)} + ...\\
&v_x = \epsilon v_x^{(1)} + \epsilon^2 v_x^{(2)} + ...\\
&p = p_0 + \epsilon p^{(1)} + ...\\
&\varepsilon = \varepsilon_0 + \epsilon\varepsilon^{(1)} + ...
\end{aligned} \tag{26}$$

We now use the stretched coordinates (25) and expansions (26) in (15), (16) and (17). From the lowest order of ϵ, we obtain:

$$A_\lambda \frac{\partial}{\partial\xi}\begin{pmatrix} \tilde{v}^{(1)} \\ \tilde{B}^{(1)} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{27}$$

where

$$A_\lambda = \begin{pmatrix} -\lambda & -\frac{v_A^2}{B_0} \\ -B_0 & -\lambda \end{pmatrix} \tag{28}$$

and v_A is the Alfven velocity which can be derived as:

$$v_A^2 = \frac{B_0^2}{\mu\varepsilon_0} \tag{29}$$

Also from the equation (28) we have:

$$\lambda = v_A \qquad , \qquad \tilde{v}^{(1)} = -\frac{v_A}{B_0}\tilde{B}^{(1)} \tag{30}$$

At the order of ϵ^2, we can write:

$$-\frac{\lambda}{\rho_{B0}}\frac{\rho_B^{(1)}}{\partial\xi} + \frac{\partial v_x^{(1)}}{\partial\xi} + \frac{\partial v_y^{(1)}}{\partial\eta} + \frac{\partial v_z^{(1)}}{\partial\zeta} = 0 \tag{31}$$

$$\lambda\varepsilon_0\frac{\partial v_x^{(1)}}{\partial\xi} = \frac{27g_h^2}{8m_G^2}\rho_{B0}\frac{\rho_B^{(1)}}{\partial\xi} + \frac{1}{\mu_0}\left[B_y^{(1)}\frac{\partial B_y^{(1)}}{\partial\xi} + B_z^{(1)}\frac{\partial B_z^{(1)}}{\partial\xi}\right] \tag{32}$$

$$\lambda\frac{\partial B_x^{(1)}}{\partial\xi} = B_0\frac{\partial v_y^{(1)}}{\partial\eta} + B_0\frac{\partial v_z^{(1)}}{\partial\zeta}. \tag{33}$$

So that the equations (31) and (33) result:

$$\frac{\rho_B^{(1)}}{\rho_{B0}} = \frac{v_x^{(1)}}{v_A} + \frac{B_x^{(1)}}{B_0} \tag{34}$$

and

$$\frac{\partial B_x^{(1)}}{\partial\xi} + \nabla_\perp.\mathbf{B}_\perp^{(1)} = 0 \tag{35}$$

where

$$\nabla_{\perp} = (\frac{\partial}{\partial \eta}, \frac{\partial}{\partial \zeta}) \qquad , \qquad \mathbf{B}_{\perp} = (B_y, B_z). \tag{36}$$

Also, from (32) and (34) we arrive at:

$$v_x^{(1)} = \frac{v_A}{v_A^2 - v_S^2} \left[v_S^2 \frac{B_x^{(1)}}{B_0} + \frac{v_A^2}{2} \frac{\mid \tilde{B}^{(1)} \mid^2}{B_0^2} \right] \tag{37}$$

where v_S is defined as:

$$v_S^2 = \frac{27 g_h^2}{8 m_G^2} \frac{\rho_{B0}^2}{\varepsilon_0} \tag{38}$$

From the terms of the order $\epsilon^{5/2}$ and using (30), (34) and (37), the following equations can be written:

$$\frac{\partial \tilde{v}^{(1)}}{\partial \tau} - v_A \frac{\partial \tilde{v}^{(2)}}{\partial \xi} - \frac{v_A^2}{B_0} \frac{\partial \tilde{B}^{(2)}}{\partial \xi} - \frac{(1 - v_S^2) v_A^4}{B_0 (v_A^2 - v_S^2)} \left(\frac{B_x^{(1)}}{B_0} + \frac{|\tilde{B}^{(1)}|^2}{2B_0^2} \right) \frac{\partial \tilde{B}^{(1)}}{\partial \xi} =$$
$$-\tilde{\nabla} \left[\frac{v_A^4}{v_A^2 - v_S^2} \left(\frac{B_x^{(1)}}{B_0} + \frac{|\tilde{B}^{(1)}|^2}{2B_0^2} \right) \right] + i \frac{3 \alpha m_s}{\mu |Q_e|} \frac{\partial^2 \tilde{B}^{(1)}}{\partial \xi^2} \tag{39}$$

and:

$$\frac{\partial \tilde{B}^{(1)}}{\partial \tau} - B_0 \frac{\partial \tilde{v}^{(2)}}{\partial \xi} - v_A \frac{\partial \tilde{B}^{(2)}}{\partial \xi} = \frac{\partial}{\partial \xi} \left(\tilde{v}^{(1)} B_x^{(1)} - \tilde{B}^{(1)} v_x^{(1)} \right) - i \frac{3 m_s}{|Q_e|} v_A \frac{\partial^2 \tilde{v}^{(1)}}{\partial \xi^2} \tag{40}$$

$\tilde{v}^{(2)}, \tilde{B}^{(2)}, \tilde{v}^{(1)}$ and $v_x^{(1)}$ can be eliminated from equations (39) and (40) with using equations (34) and (37). Thus, we can derive the master equation containing $\tilde{B}^{(1)}$, $B_x^{(1)}$ and their derivatives as following:

$$\frac{\partial \tilde{B}^{(1)}}{\partial \tau} - B_0 \tilde{\nabla} \Upsilon + \frac{\partial}{\partial \xi} \left(\Upsilon \tilde{B}^{(1)} \right) + (1 - v_S^2) \Upsilon \frac{\partial \tilde{B}^{(1)}}{\partial \xi} + i C_1 \frac{\partial^2 \tilde{B}^{(1)}}{\partial \xi^2} = 0 \tag{41}$$

where:

$$\Upsilon = \frac{1}{2} \frac{v_A^3}{v_A^2 - v_S^2} \left(\frac{B_x^{(1)}}{B_0} + \frac{|\tilde{B}^{(1)}|^2}{2B_0^2} \right) \tag{42}$$

and

$$C_1 = \frac{3 m_s B_0}{2 \mu |Q_e|} \left(\alpha - \frac{1}{\varepsilon_0} \right) \tag{43}$$

Relations (35) and (41) are the appropriate set of equations governing the evolution of the transverse ($\tilde{B}^{(1)}$) and longitudinal ($B_x^{(1)}$) magnetic field perturbations. If we neglect the spatial variation in the transverse direction [40], the equations (35) and (41) reduce to the following single equation:

$$\frac{\partial \tilde{B}^{(1)}}{\partial \tau} + C_2 \frac{\partial}{\partial \xi} \left(|\tilde{B}^{(1)}|^2 \tilde{B}^{(1)} \right) + i C_1 \frac{\partial^2 \tilde{B}^{(1)}}{\partial \xi^2} + (1 - v_S^2) C_2 |\tilde{B}^{(1)}|^2 \frac{\partial \tilde{B}^{(1)}}{\partial \xi} = 0 \tag{44}$$

in which:

$$C_2 = \frac{1}{4B_0^2}\frac{v_A^3}{v_A^2 - v_S^2} \tag{45}$$

The above wave equation in the Cartesian coordinates (x,t) becomes:

$$\frac{\partial \hat{\tilde{B}}^{(1)}}{\partial t} + v_A \frac{\partial \hat{\tilde{B}}^{(1)}}{\partial x} + C_2 \frac{\partial}{\partial x}\left(|\hat{\tilde{B}}^{(1)}|^2 \hat{\tilde{B}}^{(1)}\right)$$
$$+iC_1 \frac{\partial^2 \hat{\tilde{B}}^{(1)}}{\partial x^2} + (1 - v_S^2)C_2 |\hat{\tilde{B}}^{(1)}|^2 \frac{\partial \hat{\tilde{B}}^{(1)}}{\partial x} = 0 \tag{46}$$

while $\hat{\tilde{B}}^{(1)} \equiv \sigma^{1/2}\tilde{B}^{(1)}$. The equation (46) describes the evolution of transverse magnetic field perturbation. The evolution of baryon density perturbation also can be derived by applying the (34) and (37) in (46). The above equation is compared with the following version of the complex Ginzburg-Landau equation:

$$\frac{\partial u}{\partial t} = iP\frac{\partial^2 u}{\partial x^2} + i\gamma u + iQ_1|u|^4 u + Q_2|u|^2\frac{\partial u}{\partial x} + Q_3 u^2 \frac{\partial \tilde{u}}{\partial x} \tag{47}$$

where all coefficients are real and u is a complex function of space and time (x,t). Equation (47) is called the derivative nonlinear Schrodinger (DNLS) equation with an additional potential, or the cubic-quintic Ginzburg-Landau equation [41]. Two terms $|u|^2\partial u/\partial x$ and $u^2\partial\tilde{u}/\partial x$ are nonlinear dispersion terms. These two terms can significantly reduce the speed of the wave pulse and deform the profile of the wave into non symmetric shapes [42]. The solution of the DNLS equation has been derived and discussed by several authors [43, 44, 45]. The equation (46) also contains two additional terms: $|\hat{\tilde{B}}^{(1)}|^2\partial\hat{\tilde{B}}^{(1)}/\partial x$ and $\partial\hat{\tilde{B}}^{(1)}/\partial x$, in comparison with the original DNLS equation [44, 45]. So, we call this equation as the modified derivative nonlinear Schrodinger (mDNLS) equation. In the next section, we derive exact solitonic solutions using the plane wave perturbation technique applied on the equation (46) [41].

5 Solitary wave solution for the mDNLS equation

In order to derive localized solutions of the (mDNLS) equation, firstly we express $\hat{\tilde{B}}^{(1)}(x,t)$ in polar coordinate as following:

$$\hat{\tilde{B}}^{(1)}(x,t) = a(x,t)e^{i\theta(x,t)} \tag{48}$$

where $a(x,t)$ and $\theta(x,t)$ are real functions. Substituting (48) into (46) and separating real and imaginary parts gives the following equations:

$$a\frac{\partial\theta}{\partial t} + C_1\left(\frac{\partial^2 a}{\partial x^2} - a(\frac{\partial\theta}{\partial x})^2\right) + v_A a\frac{\partial\theta}{\partial x} + (2 - v_S^2)C_2 a^3\frac{\partial\theta}{\partial x} = 0$$
$$-\frac{\partial a}{\partial t} + C_1\left(2\frac{\partial\theta}{\partial x}\frac{\partial a}{\partial x} + \frac{\partial^2\theta}{\partial x^2}\right) - v_A\frac{\partial a}{\partial x} - (4 - v_S^2)C_2 a^2\frac{\partial a}{\partial x} = 0 \tag{49}$$

The Stokes solution of the above equations is obtained as:

$$\hat{\tilde{B}}^{(1)}(x,t) = a_0 e^{i\left[l_0 x - ((2-v_S^2)C_2 l_0 a_0^2 - C_1 l_0^2 + v_A)t\right]}. \tag{50}$$

that we use following expressions for parameters a and θ in (48):

$$a = a_0 \qquad , \qquad \theta = l_0 x - q_0 t \tag{51}$$

where $q_0 = (2 - v_S^2)C_2 l_0 a_0^2 - C_1 l_0^2 + v_A l_0$ in which a_0 and l_0 are real constants. In the next step, we find a solution for the system of equations (49) by pretreating the nontrivial solutions (50) as follows:

$$\begin{aligned} a(x,t) &= a_0 + \alpha(x - vt = \vartheta) \\ \theta(x,t) &= \Psi(\vartheta) - (q_0 - l_0 v)t \end{aligned} \tag{52}$$

where v is a constant parameter. Inserting (52) into (49) we get:

$$\begin{aligned} &(v_A - v)a\Psi' + C_1\left(\alpha'' - a(\Psi')^2\right) + (2 - v_S^2)C_2 a^3 \Psi' + (l_0 v - q_0)a = 0 \\ &(v - v_A)\alpha' + C_1\left(2\alpha'\Psi' + a\Psi''\right) - (4 - v_S^2)C_2 a^2 \alpha' = 0 \end{aligned} \tag{53}$$

Multiplying the second equation of the set (53) by a and integrating the resulting equation yields:

$$\Psi' = \frac{K_1}{C_1 a^2} + \frac{(4 - v_S^2)C_2}{4C_1}a^2 - \frac{v - v_A}{2C_1} \tag{54}$$

where K_1 is a constant of integration. Substituting the above equation into the first equation (53) we obtain:

$$\begin{aligned} a'' = \; & \frac{1}{4C_1^2}\left[K_1 C_2(6v_S^2 - 16) - (v - v_A)^2 - 4C_1(l_0 v - q_0)\right] a \\ & + \frac{C_2}{2C_1^2}(2 - v_S^2)(v - v_A)a^3 + \frac{K_1^2}{C_1^2 a^3} + \frac{C_2^2(4 - v_S^2)(12 - 5v_S^2)}{4C_1^2}a^5 \end{aligned} \tag{55}$$

and consequently:

$$\begin{aligned} a'^2 = \; & \frac{1}{4C_1^2}\left[K_1 C_2(6v_S^2 - 16) - (v - v_A)^2 - 4C_1(l_0 v - q_0)\right] a^2 - \frac{K_1^2}{C_1^2 a^2} \\ & + \frac{(2 - v_S^2)C_2}{4C_1^2}(v - v_A)a^4 + \frac{K_2}{4} + \frac{C_2^2(4 - v_S^2)(12 - 5v_S^2)}{12C_1^2}a^6 \end{aligned} \tag{56}$$

where K_2 is a constant of integration. By setting $a^2 = \chi$, we obtain an elliptic ordinary differential equation as following:

$$\chi'^2 = \frac{-4K_1^2}{C_1^2} + K_2\chi + D\chi^2 + E\chi^3 + F\chi^4 \tag{57}$$

where:

$$\begin{aligned} D &= \frac{2K_1 C_2(3v_S^2 - 8) - (v - v_A)^2 - 4C_1(l_0 v - q_0)}{C_1^2} \\ E &= \frac{(2 - v_S^2)C_2}{C_1^2}(v - v_A) \qquad , \qquad F = \frac{C_2^2(4 - v_S^2)(12 - 5v_S^2)}{3C_1^2} \end{aligned} \tag{58}$$

Now, we can examine the behaviour of solitary solutions for this equation by considering fixed values for free parameters. When $K_1 = K_2 = 0$, the equation (57) becomes:

$$\frac{\chi'^2}{\chi^2} = D + E\chi + F\chi^2 \tag{59}$$

In this case, if coefficients D ,E and F satisfy the following conditions:

$$D > 0 \qquad and \qquad E^2 - 4DF > 0 \tag{60}$$

the equation (57) embraces the following solitonic solution:

$$\chi(\vartheta) = \chi^{\pm}(\vartheta) = \frac{2D}{-E \pm \sqrt{E^2 - 4DF}\cosh\left(\sqrt{D}\vartheta\right)} \tag{61}$$

Since $a^2 = \chi$ and $x - vt = \vartheta$, from (48) we can write :

$$\mid \hat{\tilde{B}}^{(1)}(x,t) \mid^2 = \frac{2D}{-E \pm \sqrt{E^2 - 4DF}\cosh\left(\sqrt{D}(x - vt)\right)} \tag{62}$$

This equation clearly shows that the transverse magnetic field perturbation (and consequently baryon density perturbation) propagates as stable solitary waves in cold QGP environment. The constraint $D > 0$ leads to the $(v - v_A)^2 + 4C_1(l_0 v - q_0) < 0$, so that we can determine the propagation velocity of solitary wave perturbation. It may be noted that selecting the free parameters a_0, l_0 causes a convenient range for speed of localized wave that is greater than the Alfven velocity.

The electric field perturbation can be obtained by using the Maxwell equation (3) as following. we find that the direction of this perturbation is along the initial magnetic field (x direction).

$$\hat{E}_x^{(1)}(x,y,z,t)^2 = \pm\frac{z^2y^2}{y^2 - z^2}\frac{D^{3/2}v\sqrt{E^2 - 4DF}\sinh\left(\sqrt{D}(x - vt)\right)}{\left(-E \pm \sqrt{E^2 - 4DF}\cosh(\sqrt{D}(x - vt))\right)^2} \tag{63}$$

As an interesting result, existence of this electric field causes a separation between plasma components according to their charges along the magnetic field. This means that we expect to have an electric dipole moment in the core of dense and compact astrophysical objects.

6 Discussion

A comprehensive study on the features of mentioned perturbation waves can be carried out here. The derived localized solution helps us to describe effects of different parameters of the medium on the characteristics of solitary waves propagating in the medium, based on the available information about the quark matter. Equations (62) indicate that the wave phase speed, amplitude and the width of magnetic field perturbations are functions of two important parameters ε_0 (or mass density) and $|B_0|$. For a given value of quark matter information, we have chosen $\rho_0 = 3\times10^{17} kg/m^3 \sim 169 MeV/fm^3$ and $|B_0| = 10^{10}T$ [24, 46]. Also, we take the typically amount for $\mu_0 = 4\pi \times 10^{-7} H/m$ [47], $m_G = 600 MeV$, $gh = 0.05$ and $\rho_{B0} = 0.2 fm^{-3}$. Considering above mentioned values, we can obtain v_A (from 29) , C_1 (by 43) , C_2 (using 45) and consequently other parameters.

At first, we pay attention to the propagation velocity of solitary wave perturbation. If the free parameters of the solution are taken as: $a_0 \sim 1$ and $l_0 \sim -1$, the constraint $(v - v_A)^2 + 4C_1(l_0 v - q_0) < 0$ determines a convenient range for the solitary wave speed, which is definitely greater than the Alfven velocity. After selecting a specific value for

Table 1: Alfven wave velocity and the range of solitary wave speed by considering different value of background magnetic field $|B_0|$ at$\rho_0 = 3.0 \times 10^{17}(kg/m^3)$.

$B_0(T)$	Alfven wave velocity (m/s)	The range of solitary wave velocity (m/s)
1.0×10^9	1629.088	1767.906 − 1767.919
4.0×10^{10}	66792.609	72472.161 − 72469.714
6.0×10^{10}	99374.369	107815.369 − 107869.879
4.0×10^{11}	6.532×10^5	$7.076 \times 10^5 - 7.104 \times 10^5$
6.0×10^{11}	9.791×10^5	$1.052 \times 10^6 - 1.061 \times 10^6$
1.0×10^{12}	1.63×10^6	$1.763 \times 10^6 - 1.776 \times 10^6$

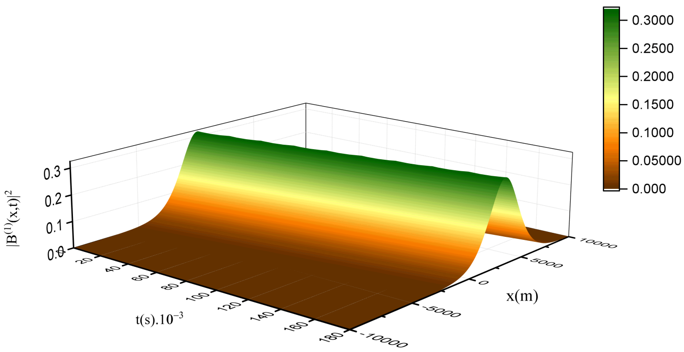

Figure 1: Time evolution of the transverse magnetic field perturbation in a cold quark matter with $\rho_0 = 3 \times 10^{17} kg/m^3$ and $|B_0| = 4 \times 10^{11} T$.

the velocity within the range of allowed values, the width and amplitude of the localized wave are calculated. Therefore, all properties of solitary wave are functions of background mass density ρ_0 and magnetic field $|B_0|$ as characteristic parameters of the quark matter environment. Derived relations show that the Alfven velocity v_A increases as the value of $|B_0|$ increases. In the table 1, we write down the acceptable rang of solitary wave speed by considering different value of background magnetic field $|B_0|$ which have been calculated using the constraint $D > 0$. In our solutions, both Alfven wave speed (v_A) and perturbation wave velocity are greater than the sound wave speed and it is an acceptable result.

Figure 1 demonstrates the time evolution of transverse magnetic field perturbation which propagates without any distortion in its initial direction. It is clear that such waves are able to reach the border of the medium and create measurable effects at the boundaries. The velocity of solitary wave has been taken as $v = 7.08 \times 10^5 (m/s)$ by considering the Alfven velocity $v_A = 6.53 \times 10^5 (m/s)$.

As mentioned before, the width and the amplitude of this solitary wave are completely dependent on the background mass density ρ_0 and magnetic field $|B_0|$. According to the

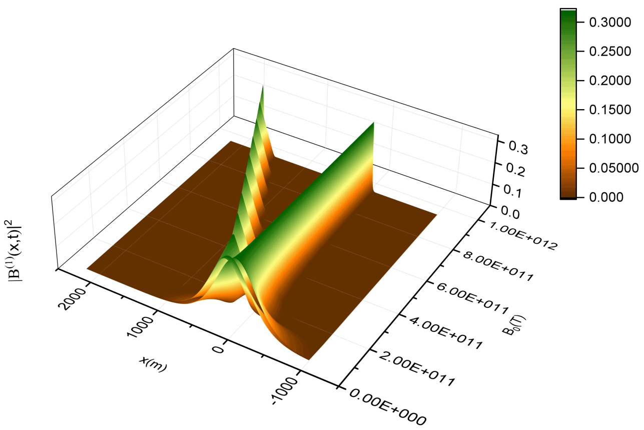

Figure 2: Soliton profiles of the transverse magnetic field perturbation at $t = 0$ and $t = 10^{-3}$ for different values of background magnetic field $|B_0|$ by considering $\rho_0 = 3 \times 10^{17} kg/m^3$.

figure 2 with the same amount of mass density, the width of solitonic profiles decreases but it propagates faster when the strength of magnetic field increases. Consequently, such localized waves can be created in a specific range of magnetic field $|B_0|$. This figure also clearly indicates that the amplitude of the solitary wave is not significantly changed and it is almost constant.

and finally, the effect of varying mass density is shown in figure 3. This figure indicates that in spite of decreasing the wave phase speed, its width increases as the mass density is given rise. Although, the width and the amplitude of the soliton is not very sensitive to the changes of mass density.

7 Conclusions and remarks

In this work, we have focused on the magneto hydrodynamic equations in an ideal and non relativistic framework for cold quark matter (QGP). Since the magnetic field oblige different trajectories and consequently different velocity for each specific, the cold matter is considered as a multi-fluid environment which consist of three flavors of quarks (u, d, s) and we could write a complete set of MHD equation with nonlinear and dispersive terms which play essential roles in the dynamics of the system. Substituting an equation of state (EOS) in MHD equations is necessary and the mQCD model is applied which gives a good description of quark matter at the present of magnetic field. Because a perfect and analytical solution of these equations is not feasible, the behavior of small amplitude perturbation of variables such as densities, velocities and magnetic field have been investigated by applying the reductive perturbation method (RPM). The result is the modified derivative nonlinear Schrodinger (mDNLS) equation which governs the mass density (localized transverse magnetic field) perturbation waves. Also, Creating a transverse magnetic field perturbation induced an electric field perturbation in the direction of initial magnetic field which can caused a separation of

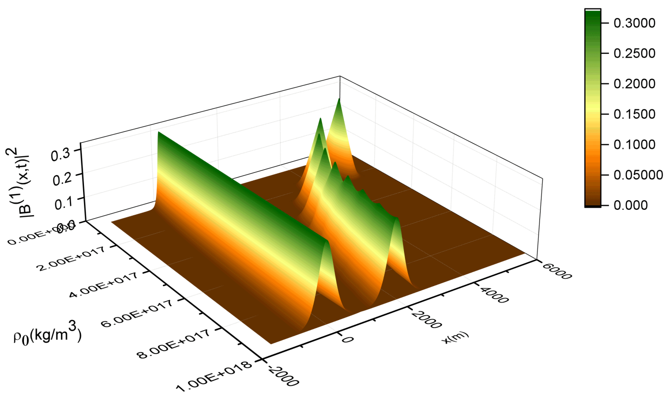

Figure 3: Soliton profiles of the transverse magnetic field perturbation at $t = 0$ and $t = 10^{-3}$ for different values of background mass density ρ_0 by considering $|B_0| = 10^{10}T$.

components according to their charges along the magnetic field.

These solitary waves can be propagated without any distortion in its initial direction in cold quark matter. The width and the velocity of these localized waves are functions of the background mass density and magnetic field. The solitonic wave phase speed increases as the magnetic field B_0 increases, while it decreases by increasing values of the mass density ρ_0. Also, increasing the magnetic field B_0 reduces the width of solitary waves; but variation of the mass density has not significant effects in the characteristics of solitary waves. Therefore, we can conclude that the solitary profiles are expected to be established in such media when the background magnetic field is in a specific range.

The significant result of this article is that, the small amplitude propagation of localized waves in cold quark matter is not governed by the KdV equation and they are solitons of the modified DNLS equation which behave very different from the KdV localized solutions. Derived equations also clearly indicate that solitary waves in cold quark matter (by considering its electromagnetic effects) are completely stable, independent from the existence of Laplacian terms in the energy density (which are generally weak).

Similar investigation can be done for neutron stars using other proper EOS like [48]. It is expected that localized perturbation in magnetic field and energy density also appears in this media. In other word, it is expected that presented behavior may be observed in different structures of compact astrophysical objects.

So, there are many works in this subject which should be done (or revised). It is possible that there exist some sorts of instabilities in the propagating waves as solutions of DNLS equations, which should be investigated. Same problem but with different models for the equation of state can be solved, and results should be compared. The problem also is open for other forms of super dense media, like hadronic gas and nuclear matter as expected to be found in compact astrophysical objects.

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