Iranian Journal of Astronomy and Astrophysics
Vol. 4, No. 3, Autumn 2017
©Available online at http://journals.du.ac.ir
DOI:10.22128/ijaa.2017.129

Iranian Journal of Astronomy and Astrophysics

Thermal measurement induced disturbance in the Bose-Hubbard Hamiltonian

Hamed Jafarzadeh[1] · Hakimeh Jaghouri[2]
[1] Quchan University of Technology, Quchan, Iran
[2] Department of Physics, Ferdowsi University of Mashhad, 91775-1436, Mashhad, Iran

Abstract. We study measurement induced disturbance (MID) in a qutrit -qutrit system with considering the effect of the external magnetic field, nonlinear and linear coupling constants and temperature. We show that all of these parameters have effective roles in MID. We also investigate the effect of finite external magnetic fields direction as parallel and anti-parallel on MID, and find some interesting results.

Keywords: qutrit, measurement induced disturbance, Bose-Hubbard Hamiltonian

1 Introduction

Entanglement is fundamental property which makes distinct quantum world from classical world in physics. The entanglement of two distant particles, which distinctly shows the non local nature in Quantum Mechanics, is provided particles in astrophysical objects [1]. It is shown that quantum mechanics plays an important role for cosmological neutrinos. These neutrinos could be considered as a nonlocalisable ensemble of entangled particles [2, 3]. For quantifying quantum correlation, measures as quantum discord and MID are introduced, which are nonzero for some separable states, so they includes entanglement [4, 5]. Quantum discord has an optimization process which could be just calculated analytically for a few qubit-qubit systems. MID is used to distinguish quantum correlations from classical ones. The calculation of MID does not need optimization, thus MID is a simple way to evaluate the quantum correlation. It is clear that external magnetic field plays an important role on the quantum correlations and entanglement in spin systems such as Bose-Hubbard, Heisenberg and Ising models [6, 7]. Recently, we investigate the effect of different system parameters on the thermal geometric discord and entanglement [6]. Entanglement against temperature is more fragile than quantum discord. Entanglement and quantum correlation have been studied for spin half (qubit-qubit) systems a lot while for spin one (qutrit-qutrit) systems, there are a few studies [11, 12, 13, 14, 15, 17, 18]. The motivation of this work is a comparison of quantum correlations which are measured by MID and geometric quantum discord that are evaluated in Ref. [6].

This work is organized as follows. The Hamiltonian of model is introduced in section 2. In section 3, the definition of MID has been provided; then we investigate thermal MID for a qutrit-qutrit system in zero external magnetic fields due to the linear and non linear coupling constants at different temperatures. In the following, in section 4 effect of external magnetic fields has been investigated. The comparison and the result have been provided in section 5.

2 Hamiltonian of system and the solutions

In order to control the system of atoms in optical lattice (atoms in traps), progress of laser cooling and trapping has made available more ways for us. We consider a system which has two wells in the lattice with one qutrit (spin-1 atom) in each well. This lattice would be made of three perpendicular laser beams [6, 19, 20, 21, 21]. A Bose-Hubbard Hamiltonian describes this system. Each well has only one qutrit in the regime of the Mott, so the Hamiltonian of the system could be defined for finite and small hopping as [6, 19, 20, 21, 21]

$$H = \eta + J(\vec{S_1} \cdot \vec{S_2}) + K(\vec{S_1} \cdot \vec{S_2})^2, \tag{1}$$

where $\eta = J - K$, $K = -\frac{2r^2}{3U_2} - \frac{4r^2}{U_0}$ and $J = -\frac{2r^2}{U_2}$, with hopping matrix elements (r). Here $U_t(t = 0, 2)$ are the repulsion potential of Hubbard, where t is the eigenvalue of the total spin $\vec{S}$. Because η is constant, it could be ignored. The Heisenberg interaction strength (linear coupling) is J, this system is called antiferromagnetic for $J > 0$ and ferromagnetic for $J < 0$. Parameter K shows the nonlinear coupling constant. The Hamiltonian of a qutrit-qutrit system, in the presence of the nonuniform external magnetic fields along the z axis, B_m $(m = 1, 2)$, is [19, 20, 21, 21, 22]

$$H = J(\vec{S_1} \cdot \vec{S_2}) + K(\vec{S_1}.\vec{S_2})^2 + B_1 S_{1z} + B_2 S_{2z}, \tag{2}$$

where $\vec{S_m} = (S_{mx}, S_{my}, S_{mz})$ with $m = 1, 2$ are the spin operators defined as

$$S_{mx} = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}, S_{my} = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{bmatrix},$$
$$S_{mz} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}.$$

We suppose that the magnitude of B_1 and B_2 are same but they could be anti-parallel or parallel. At first, we find the eigenvalues (E_j) and the eigenstates $(|\phi_j\rangle)$ of the model.

For parallel magnetic fields, $B_1 = B_2 = B$, the eigenstates are calculated as [6].

$$\begin{aligned}
|\phi_1\rangle &= \frac{1}{\sqrt{6}}(|-1,1\rangle + 2|0,0\rangle + |1,-1\rangle), \\
|\phi_2\rangle &= \frac{1}{\sqrt{2}}(-|-1,1\rangle + |1,-1\rangle), \\
|\phi_3\rangle &= \frac{1}{\sqrt{3}}(|-1,1\rangle - |0,0\rangle + |1,-1\rangle), \\
|\phi_4\rangle &= |1,1\rangle, \\
|\phi_5\rangle &= \frac{1}{\sqrt{2}}(|0,1\rangle + |1,0\rangle), \\
|\phi_6\rangle &= \frac{1}{\sqrt{2}}(-|0,1\rangle + |1,0\rangle), \\
|\phi_7\rangle &= \frac{1}{\sqrt{2}}(-|-1,0\rangle + |0,-1\rangle),
\end{aligned}$$

$$
\begin{aligned}
|\phi_8\rangle &= \frac{1}{\sqrt{2}}(|-1,0\rangle + |0,-1\rangle), \\
|\phi_9\rangle &= |-1,-1\rangle. \qquad (3)
\end{aligned}
$$

and corresponding eigenvalues are as [6]

$$
\begin{aligned}
E_1 &= K + J, \\
E_2 &= K - J, \\
E_3 &= 4K - 2J, \\
E_4 &= 2B + K + J, \\
E_5 &= B + K + J, \\
E_6 &= B + K - J, \\
E_7 &= K - B - J, \\
E_8 &= K - B + J, \\
E_9 &= -2B + K + J \qquad (4)
\end{aligned}
$$

The density matrix for the system is

$$
\rho^{AB} = \frac{1}{Z}\begin{bmatrix}
\rho_{11} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \rho_{22} & 0 & \rho_{24} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \rho_{33} & 0 & \rho_{35} & 0 & \rho_{37} & 0 & 0 \\
0 & \rho_{42} & 0 & \rho_{44} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \rho_{53} & 0 & \rho_{55} & 0 & \rho_{57} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \rho_{66} & 0 & \rho_{68} & 0 \\
0 & 0 & \rho_{73} & 0 & \rho_{75} & 0 & \rho_{77} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \rho_{86} & 0 & \rho_{88} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho_{99}
\end{bmatrix}. \qquad (5)
$$

Where the elements of this matrix are

$$
\begin{aligned}
\rho_{11} &= e^{-\frac{\beta E_4}{T}} \qquad (6) \\
\rho_{22} &= \rho_{44} = \frac{1}{2}(e^{-\beta E_3} + e^{-\beta E_6}) \\
\rho_{33} &= \rho_{77} = \frac{1}{6}(e^{-\beta E_2} + 3e^{-\beta E_7} + 2e^{-\beta E_8}) \\
\rho_{55} &= \frac{1}{3}(2e^{-\beta E_2} + e^{-\beta E_8}) \\
\rho_{66} &= \rho_{88} = \frac{1}{2}(e^{-\beta E_9} + e^{-\beta E_1}) \\
\rho_{99} &= e^{-\frac{\beta E_5}{T}} \\
\rho_{24} &= \rho_{42} = \frac{1}{2}(e^{-\beta E_3} - e^{-\beta E_6}) \\
\rho_{35} &= \rho_{57} = \rho_{53} = \rho_{75} = \frac{1}{3}(e^{-\beta E_2} - e^{-\beta E_8}) \\
\rho_{37} &= \rho_{73} = \frac{1}{6}(e^{-\beta E_2} - 3e^{-\beta E_7} + 2e^{-\beta E_8})
\end{aligned}
$$

where $\beta = \frac{1}{k_B T}$ with $k_B = 1$ and T is temperature of the system, $Z = \sum_{j=1}^{9} e^{-\beta E_j}$ is the partition function. The eigenvalues and the eigenstates have been calculated for anti-parallel magnetic fields ($B_1 = -B_2 = B$). They did not have a compact form, so we do not mention them here.

3 Measurement induced disturbance

Calculation of quantum discord for a qutrit-qutrit system is difficult because of optimization, so with application of local measurements on state of system, one can quantify quantumness of system [5]. Spectral resolutions of the reduced density matrix ρ^A and ρ^B are respectively $\rho^A = \sum_{i=1}^{3} p_i^A \Pi_i^A$ and $\rho^B = \sum_{j=1}^{3} p_i^B \Pi_i^B$. Orthogonal projective operators which are one dimensional, for parts A and B for qutrit-qutrit system can be written as $\{\Pi_i^A\}$ and $\{\Pi_j^B\}$ with $i, j = 1, 2, 3$. These local projective measurement produce state $\Pi(\rho^{AB})$ as

$$\Pi(\rho^{AB}) = \sum_{i,j=1}^{3} (\Pi_i^A \otimes \Pi_j^B)\rho^{AB}(\Pi_i^A \otimes \Pi_j^B) \tag{7}$$

In general, a bipartite state ρ^{AB} (qutrit-qutrit system) is not classical but after local measurement $\Pi(\rho^{AB})$ is classical. MID is defined as difference between mutual information for state before and after measurements [5].

$$Q(\rho^{AB}) = I(\rho^{AB}) - I(\Pi(\rho^{AB})), \tag{8}$$

where the quantum mutual information $I(\rho^{AB})$ describes the total correlation for a bipartite state ρ^{AB} while $\Pi(\rho^{AB})$ is a classical state so $I(\Pi(\rho^{AB}))$ describes the classical correlations in ρ^{AB}. MID is the difference between the total and classical correlations. For a given state ρ^{AB} mutual information is defined as

$$I(\rho^{AB}) = S(\rho^A) + S(\rho^B) - S(\rho^{AB}). \tag{9}$$

By finding eigenvectors of reduced density matrix A and B, we get projective operators on subsystem A and B as $\Pi_i^A = \Pi_i^B = |i\rangle\langle i|$, $i = 1, 2, 3$ for subsystem A

$$\rho^A = \Pi^A(\rho^A) = \begin{bmatrix} \rho_{11}+\rho_{22}+\rho_{33} & 0 & 0 \\ 0 & \rho_{44}+\rho_{55}+\rho_{66} & 0 \\ 0 & 0 & \rho_{77}+\rho_{88}+\rho_{99} \end{bmatrix} \tag{10}$$

and for subsystem B

$$\rho^B = \Pi^B(\rho^B) = \begin{bmatrix} \rho_{11}+\rho_{44}+\rho_{77} & 0 & 0 \\ 0 & \rho_{22}+\rho_{55}+\rho_{88} & 0 \\ 0 & 0 & \rho_{33}+\rho_{66}+\rho_{99} \end{bmatrix}. \tag{11}$$

Projective operator (Π) on ρ^{AB} is

$$\Pi(\rho^{AB}) = diag\{\rho_{11}, \rho_{22}, \rho_{33}, \rho_{44}, \rho_{55}, \rho_{66}, \rho_{77}, \rho_{88}, \rho_{99}\}. \tag{12}$$

Finally, MID is calculated as

$$MID = -2\, q_1 \log_2(q_1) - q_2 \log_2(q_2) - 2\, q_3 \log_2(q_3) - 2\, q_4 \log_2(q_4) + q_5 \log_2(q_5) \tag{13}$$
$$+q_6 \log_2(q_6) + q_7 \log_2(q_7) + q_8 \log_2(q_8) + q_9 \log_2(q_9) + q_{10} \log_2(q_{10}) + \log_2(q_{11}).$$

When we have,
$q_1 = \frac{e^{-\beta E_1}+e^{-\beta E_2}+e^{-\beta E_3}}{Z}$, $q_2 = \frac{e^{-\beta E_2}+4\,e^{-\beta E_3}}{Z}$, $q_3 = \frac{e^{-\beta E_4}+e^{-\beta E_5}}{Z}$, $q_4 = \frac{e^{-\beta E_6}+e^{-\beta E_9}}{Z}$, $q_5 = 2\frac{e^{-\beta E_1}}{Z}$, $q_6 = 6\frac{e^{-\beta E_3}}{Z}$, $q_7 = 2\frac{e^{-\beta E_5}}{Z}$, $q_8 = 3\frac{e^{-\beta E_2}}{Z}$, $q_9 = 2\frac{e^{-\beta E_4}}{Z}$, $q_{10} = 2\frac{e^{-\beta E_6}}{Z}$ and $q_{11} = 2\frac{e^{-\beta E_9}}{Z}$.

Firstly, we study the effect of linear coupling constant and temperature on MID, so we assume that magnetic fields and nonlinear coupling are zero. Fig. 1 shows, at zero temperature, for $J > 0$ the ground state of system is an entangled state ($|\phi_3\rangle$). In these conditions MID is non-zero ($MID \simeq 1.6$) while for $J < 0$ the ground state of system is a mixed state as $\rho = \frac{1}{5}(|\phi_1\rangle\langle\phi_1| + |\phi_4\rangle\langle\phi_4| + |\phi_5\rangle\langle\phi_5| + |\phi_8\rangle\langle\phi_8| + |\phi_9\rangle\langle\phi_9|)$, which is a classical mixed state and MID is exactly shows quantum correlations. As temperature increases, the constant behavior of MID with respect to J changes.

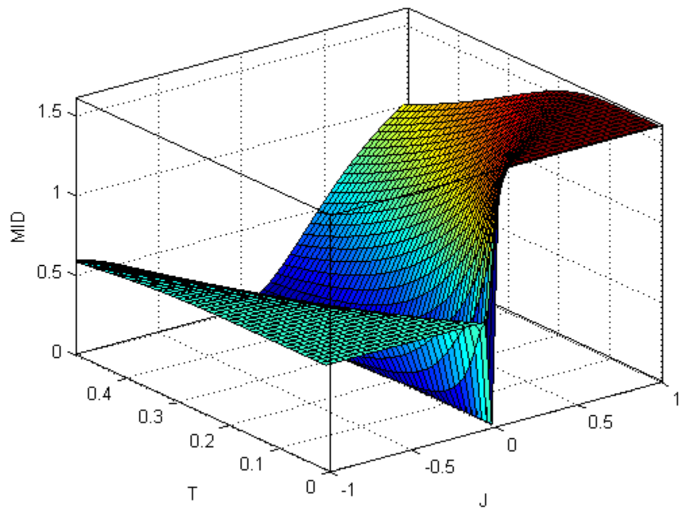
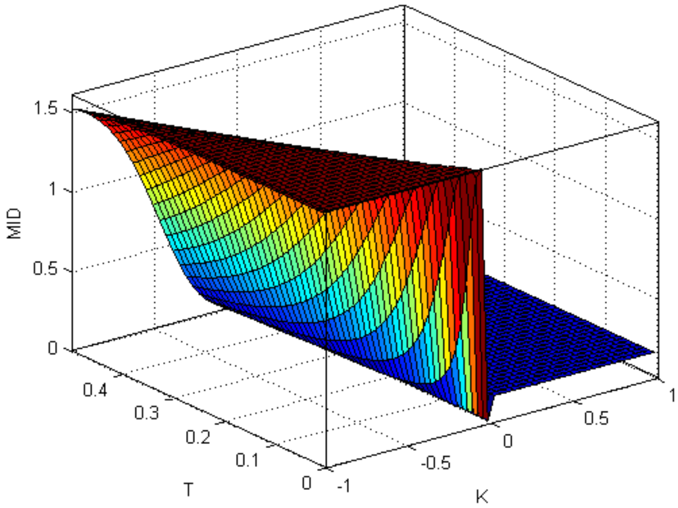


Figure 1: MID for $B_1 = B_2 = 0$ as functions of T and J for $K = 0$

In the following, we investigate the behaviors of MID as a function of K and T with zero magnetic fields and $|J| = 0$. Fig. 2 shows at $T = 0$, the sate of system, for $K > 0$, is mixed state as $\rho = \frac{1}{8}(|\phi_1\rangle\langle\phi_1| + |\phi_2\rangle\langle\phi_2| + |\phi_4\rangle\langle\phi_4| + |\phi_5\rangle\langle\phi_5| + |\phi_6\rangle\langle\phi_6| + |\phi_7\rangle\langle\phi_7| + |\phi_8\rangle\langle\phi_8| + |\phi_9\rangle\langle\phi_9|)$, and MID of this system has constant value ($MID = 0.4$) which shows quantum correlations. While this figure demonstrates that for $K < 0$, the ground state of the system is an entangled state $|\phi_3\rangle$. At zero temperature, MID is not a function of K as Fig. 2 shows. Temperature will destroy constant behavior of MID with respect to K.

It seems necessary to consider the effect of both linear and nonlinear coupling constants on MID in zero magnetic field. At zero temperature for $J > 0$ ($J \leq 0$), point of quantum phase transition is on $K = \frac{J}{3}$ ($K = J$). There is an interval of nonlinear coupling constant which MID is constant. In transition point, for $J > 0$, the state of the system will change from $|\phi_3\rangle$ to a state which has been made of combinations of $|\phi_2\rangle$, $|\phi_6\rangle$ and $|\phi_7\rangle$. While for zero linear coupling, state changes from $|\phi_3\rangle$ to a combinations of $|\phi_1\rangle$, $|\phi_4\rangle$, $|\phi_5\rangle$, $|\phi_8\rangle$ and $|\phi_9\rangle$. These results are the same with the behavior of geometric discord [6]. Fig.3 verifies above results at a finite temperature ($T = 0.2$).



a.

Figure 2: MID for $B_1 = B_2 = 0$ as functions of K and T for $J = 0$

4 Effect of external magnetic field on MID

In the this section, we study the effect of the external magnetic field (B) on MID. Moreover, it is interesting to survey effect of the direction of two external magnetic fields on MID, so we consider two cases as parallel and anti-parallel external magnetic fields on our physical system.

Fig. 4 shows MID for $J = 0.2$ and $K = -0.3$ at different temperatures as a function of magnetic field $B_1 = B_2 = B$. In this figure, one can find that MID is maximal for zero external magnetic field ($B = 0$). The effect of temperature is destructive on MID as figure shows.

At the final case, we consider quantum correlations for a system with two finite anti-parallel external magnetic fields. For this case MID, have been calculated numerically. Quantum correlations have been shown in Fig. 5 as a function of a finite external magnetic field for $K = -0.3$ and $J = 0.2$ at several different temperatures. For anti-parallel magnetic fields ($B_1 = -B_2 = B$), the figure shows that with respect to $|B|$ at first MID decreases then for big magnetic field, it is constant. For anti-parallel magnetic fields, MID is more robust than parallel magnetic fields, as Fig. 4 and 5 show, so direction of magnetic fields plays a significant role in quantum correlations.

5 Comparisons and results

In this work, we have calculated quantum correlation by MID for Bose-Hubbard system under the effect of magnetic fields at finite temperature. We find some situations with non-zero temperatures that the quantum correlation are more resistant than entanglement; this

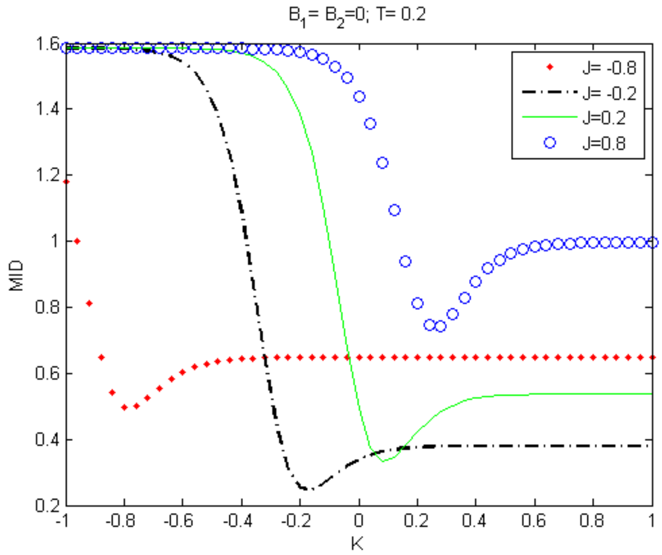


Figure 3: MID as a function of K for $T = 0.2$, $B_1 = B_2 = 0$

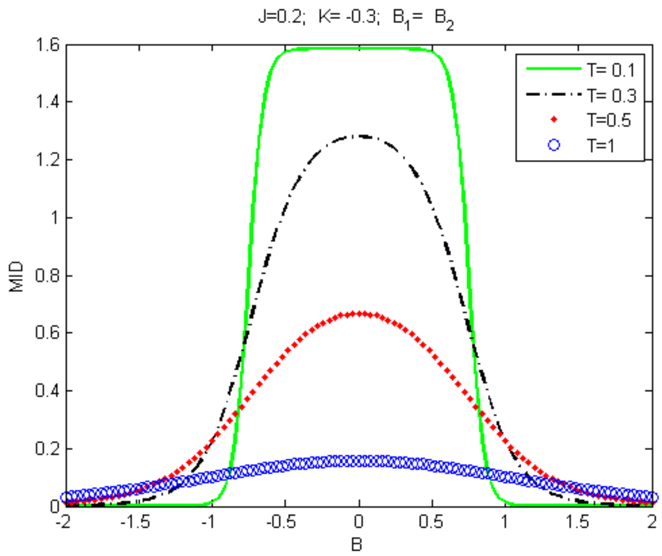


Figure 4: MID as a function of $B_1 = B_2 = B$ for $J = 0.2$ and $K = -0.3$

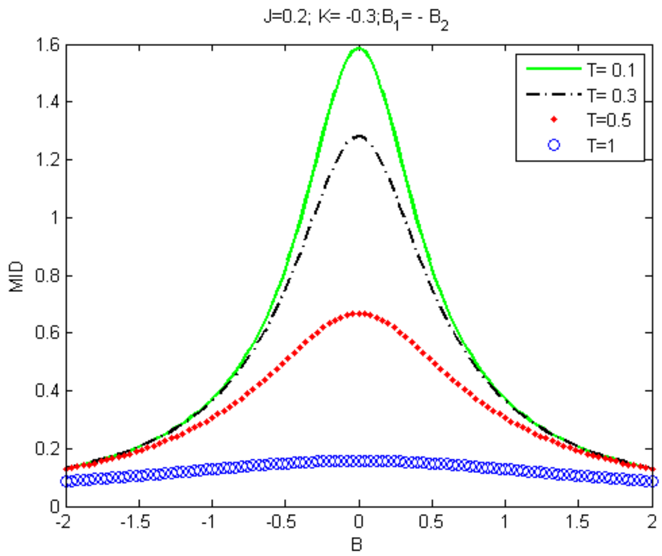


Figure 5: MID as a function of $B_1 = -B_2 = B$ for $J = 0.2$ and $K = -0.3$

is demonstrated in Figs. 1 and 2. In Ref. [6], the quantum correlations and the entanglement have been investigated by exact geometric discord and negativity. We show for some values of J and K, that the direction of magnetic field can amplify MID as geometric discord and the entanglement [6]. The general behavior of MID in the presence of parallel magnetic fields ($B_1 = B_2 = B$) is similar to the zero magnetic field. While for anti-parallel magnetic field ($B_1 = -B_2 = B$), the behavior of MID is very different duo to the dependence of eigenstates on B, as shown in Fig. 6. In this condition, magnetic field could strengthen MID as Fig. 6 demonstrates which is an interesting result.

We have shown in Fig. 5 at a high temperature ($T = 1$) and small magnetic fields ($|B| < 0.37$), MID and the geometric discord behave similarly while negativity behaves differently. In this situation, the MID and geometric discord are non-zero in contrast to zero negativity.

Acknowledgment This work was supported by Quchan University of Technology under a grant number 10799.

References

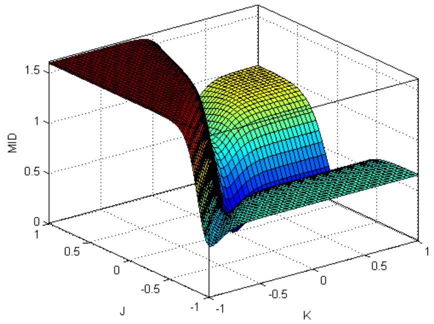
[1] Gmez, J., Peimbert, A., Echevarra, J.: Optical quantum entanglement in astrophysics. Revista mexicana de astronoma y astrofsica, **45**, 179-189 (2009).

[2] Pitknen, M.: Magnetospheric consciousness (2018).

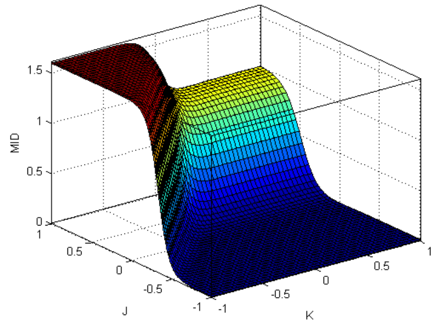
[3] Giaccari, S., Modesto, L., Rachwa, L., Zhu, Y.: Finite entanglement entropy of black holes. The European Physical Journal C, **78**, 459 (2018).

[4] Ollivier, H., Zurek, W.H.: Quantum discord: a measure of the quantumness of correlations. Phys. Rev. Lett. **88**, 017901 (2001)

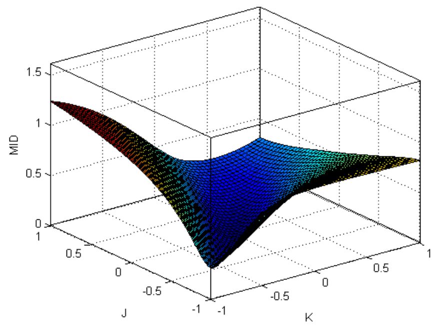
[5] Luo, S.: Using measurement-induced disturbance to characterize correlations as classical or quantum. Phys. Rev. A , **77**, 022301 (2008).



a1.



a2.



b1.

Figure 6: Geometric discord and Negativity as functions of K and J at $T = 0.2$ for a: $B_1 = B_2 = 0$, b: $B_1 = B_2 = 1$ and c: $B_1 = -B_2 = 1$

[6] Jaghouri, H., Nazifkar, S., Jafarzadeh, H., Javidan, K.: Thermal quantum correlation and entanglement in the BoseHubbard Hamiltonian. Quantum Information Processing, **17**, 284 (2018).

[7] Jaghouri, H., Sarbishaei, M., Javidan, K.: Thermal entanglement and lower bound of the geometric discord for a two-qutrit system with linear coupling and nonuniform external magnetic field. Quantum Inf Process. **16**, 124 (2017)

[8] Zou, H. M., Fang, M. F.: Discord and entanglement in non-Markovian environments at finite temperatures. Chin. Phys. B **25**, 090302 (2016)

[9] You-Neng, G., Mao-Fa, F., Xiang, L., Bai-Yuan, Y.: Dynamics of quantum discord in a two-qubit system under classical noise. Chin. Phys. B **23**, 034204 (2014)

[10] Werlang, T., Souza, S., Fanchini, F. F., Boas, C. V.: Robustness of quantum discord to sudden death. Phys. Rev. A **80**, 024103 (2009)

[11] Jaghouri, H., Sarbishaei, M., Javidan, K.: Evolution of entropy in different types of non-Markovian three-level systems: Single reservoir vs. two independent reservoirs. Pramana, **86**, 997 (2016)

[12] Guo, Y. N., Fang, M. F., Wang, G. Y., Zeng, K.: Distillability sudden death and sudden birth in a two-qutrit system under decoherence at finite temperature. Quantum Inform. Process. **15**, 2851 (2016)

[13] Jiang, H., Mao-Fa, F., Bai-Yuan, Y., Xiang, L.: Distillability sudden death in a two qutrit systems under a thermal reservoir. Chin. Phys. B **21**, 084205 (2012)

[14] Caves, C. M., Milburn, G. J.: Qutrit entanglement. Optics communications, **179**, 439 (2000)

[15] Xiao, X., Li, Y. L.: Protecting qutrit-qutrit entanglement by weak measurement and reversal. Eur. Phys. J. D. **67** 204 (2013)

[16] Yan, X. Q., Liu, G. H., Chee, J.: Sudden change in quantum discord accompanying the transition from bound to free entanglement. Phys. Rev. A **87**, 022340 (2013)

[17] Yuan, Y. L., Hou, X. W.: Thermal geometric discords in a two-qutrit system. Int. J. of Quantum Inform. **14**, 1650016 (2016)

[18] Hou, X. W., Lei, X. F., Chen, B.: Thermal quantum and classical correlations in a two-qutrit system. Eur. Phys. J. D **67**, 1 (2013)

[19] Greiner M., Mandel O., Esslinger T., Hnsch TW., Bloch I.: Quantum phase transition from a superfluid to a Mott insulator in a gas of ultracold atoms. Nature, London **415** (2002)

[20] Zhang, G. F., Li, S. S.: The effects of nonlinear couplings and external magnetic field on the thermal entanglement in a two-spin-qutrit system. Opt. Commun. **260**, 347 (2006)

[21] Zhang, G. F., Li, S. S., Liang, J. Q.: Thermal entanglement in Spin-1 biparticle system. Opt. Commun. **245**, 457 (2005)

[22] Yip, S. K.: Dimer state of spin-1 bosons in an optical lattice. Phys. Rev. Lett. **90** 250402 (2003)