Journal of Holography Applications in Physics
Volume 3, Issue 1, Winter 2023, 71-79
© Available online at http://jhap.du.ac.ir
DOI: 10.22128/jhap.2023.653.1040
Online ISSN: 2783-3518

Journal of Holography Applications in Physics

Regular article

Electromagnetic structure of the nucleon and the Roper resonance in hard wall AdS/QCD model

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Received: January 25, 2023; Revised: March 03, 2023; Accepted: March 20, 2023

Abstract. The electromagnetic structure of the nucleon and the second radial excitation of the nucleon is studied using hard-wall AdS/QCD model. We plot the Dirac, and Pauli form factors and helicity amplitudes dependencies on transferred momentum square. $\mathrm{N}+\gamma^{*} \rightarrow \mathrm{R}(1710)$ transition electromagnetic form factors and both the transverse $A_{1/2}$ and longitudinal $S_{1/2}$ helicity amplitudes are calculated resulting in good agreement with data, with the MAID parametrization and nonrelativistic quark model at some point.

Keywords: AdS/CFT Correspondence; Hard-Wall Model; Roper Nucleon; Electromagnetic Form Factors; Pauli and Dirac Form Factors; Helicity Amplitude.

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1 Introduction

Understanding the internal structure of the nucleon as a composite system, built up of quarks and gluons, is one of the most important unsolved problems in hadron physics. Studying the internal structure of nucleons plays an important role in solving this problem. Several theoretical models have been proposed to interpret the nucleon resonance spectrum and the information associated with its internal structure.

The Roper resonance was first considered in the context of AdS/QCD in Refs. [1-4], where the Dirac form factor for the electromagnetic nucleon-Roper transition was calculated in light-front holographic QCD.

Nucleon resonances have also been discussed in the AdS soft-wall approach in Refs. [1-4]. In Ref. [1] the Dirac form factor for the electromagnetic nucleon-Roper transition was calculated in light-front holographic QCD. In addition, in Ref. [2], a formalism for the study of all nucleon resonances in soft-wall AdS/QCD has been proposed and a detailed description of Roper-nucleon transition properties (form factors, helicity amplitudes and transition charge radii) was performed.

The data extracted from experiments in facilities such as Jefferson Lab (JLab) and MAMI (Mainz), allow us to extract the electromagnetic transition form factors of resonant states in the region $Q^2 < GeV^2 (Q^2 = -q^2)$, corresponding to the first and second resonance region $W < 1.6 GeV$, with W being the $\gamma^* N$ invariant mass [5,6]. The planned JLab 12-GeV upgrade will enable us to make a detailed study of the resonances in the third-resonance region (W ≈ 1.7 GeV) [6, 7].

$\mathrm{N} + \gamma^* \to \mathrm{R}(1535)$ transition form factors are studied as a negative parity resonance within the covariant spectator quark model [8]. In Ref. [4] authors presented a description of electromagnetic properties of the nucleon and the Roper at small finite temperatures using the formalism developed in Ref. [9].

In this work we discuss the $\mathrm{N}\gamma^* \to \mathrm{N}^*$ transition electromagnetic form factors for resonances N^* with positive parity using hard-wall AdS/QCD model.

The paper is organized as follows. In Sec.2 we introduce the profile function for nucleons and vector field. In Sec.3 we present the analytical calculation and the numerical analysis of electromagnetic form factors and helicity amplitudes of the nucleon and the Roper framework soft-wall AdS/QCD model. Finally, Sec.4 contains our numerical results and conclusions.

2 Profile functions for nucleons and vector field

As known, the minimal bulk action for the spinor field is written as follows:

$$S_N = \int d^5x \sqrt{g}(i\bar{N}_1 e_A^M \Gamma^A D_M N_1 - m_5 \bar{N}_1 N_1), \tag{1}$$

where g- is the determinant of the AdS metric, Γ^A –are the Dirac matrices. The Lorentz and gauge-covariant derivative is defined in the below form:

$$D_M = \partial_M - \frac{i}{4}\omega_{M\,BC}\Sigma^{BC}. \tag{2}$$

Equation of motion obtained from the action (1) has an explicit form:

$$(iz\eth + z\gamma^5\partial_z - 2\gamma^5 - m_5)N_1 = 0. \tag{3}$$

Then the Dirac equation (3) will be written as equations for the profile functions $F_{1,2,L,R}$:

$$\left(\partial_z^2 - \frac{4}{z}\partial_z + \frac{(6 - m_5 - m_5^2)}{z^2}\right) f_{1R}(p,z) = -|P|^2 f_{1R}(p,z), \tag{4}$$

$$\left(\partial_z^2 - \frac{4}{z}\partial_z + \frac{(6 + m_5 - m_5^2)}{z^2}\right) f_{1L}(p,z) = -|P|^2 f_{1L}(p,z). \tag{5}$$

Solutions to the equations (4), (5) are expressed in terms of Bessel functions $J_{2,3}$ [10,11]:

$$f_{1L}^n(z) = c_1^n z^{\frac{5}{2}} J_2(m_n z), \tag{6}$$

$$f_{1R}^n(z) = c_1^n z^{\frac{5}{2}} J_3(m_n z), \tag{7}$$

$$f_{2L}^n(z) = -c_2^n z^{\frac{5}{2}} J_3(m_n z), \tag{8}$$

$$f_{2R}^n(z) = c_2^n z^{\frac{5}{2}} J_2(m_n z), \tag{9}$$

where $c_{1,2}$ are constants were found from the normalization conditions as below:

$$c_{1,2} = \left|\frac{\sqrt{2}}{z_m J_2(m_n z_m)}\right|. \tag{10}$$

Action for the 5-dimensional vector field VM corresponding to the photon field in the boundary theory is written in the form:

$$S_V = -\frac{1}{2g_5^2}\int d^5x \sqrt{g}\; Tr F_{MN}^2. \tag{11}$$

If the Fourier transform of the vector field is written in (11) action, the equation of motion can be expressed as follow for bulk-to-boundary propagator:

$$\partial_z\left(\frac{1}{z}\partial_z V(q,z)\right) + \frac{q^2}{z} V(q,z) = 0. \tag{12}$$

The solution of the equation (12) is written in terms of the Bessel function [12]:

$$V(q,z) = \frac{\pi}{2} zq \left(\frac{Y_0(q,z_0)}{J_0(q,z_0)} J_1(qz) - Y_1(qz) \right). \tag{13}$$

3 Electromagnetic form factors. Helicity amplitudes

The action must be written to obtain electromagnetic form factors of the $N + \gamma^* \to R$ transition in bulk AdS spacetime,

$$S_{int} = \int_0^{z_m} d^4x dz \sqrt{g} L_{int}(x,z), \tag{14}$$

where $L_{int}(x,z)$ is the interaction lagrangian of two fields (holographically corresponding to hadron nucleon and the Roper nucleon) and vector field (holographical corresponding to the electromagnetic field):

$$L_{int}(x,z) = \sum_{i=+,+;\tau} c_\tau^{RN} \underline{\psi}_{i,\tau}^R(x,z) \hat{V}_i(x,z) \psi_{i,\tau}^N(x,z), \tag{15}$$

$$\hat{V}_\pm(x,z) = \tau_3 \Gamma^M V_M(x,z) \pm \frac{i}{4} \eta_V [\Gamma^M \Gamma^N] V_{MN}(x,z) \pm g_V \tau_3 \Gamma^M i \Gamma^z V_M(x,z). \tag{16}$$

According to AdS/CFT correspondence generating function is equal exponent of classical bulk action S_{int}:

$$Z_{AdS} = e^{iS_{int}}. \tag{17}$$

The vacuum expectation value of the nucleon's vector current is defined as below in holographic principle

$$\langle J_\mu \rangle = -i \frac{\delta Z_{QCD}}{\delta V_\mu(q)} |_{V_\mu = 0} = -i \frac{\delta e^{iS_{int}}}{\delta V_\mu(q)} |_{V_\mu = 0}. \tag{18}$$

Electromagnetic form factors for the $N + \gamma^* \to R(1710)$ reaction will be written in terms of integrals over the z variable from the comparison of the electromagnetic current of the Roper-nucleon transitions with the nucleon vector current [13]:

$$G_1(Q^2) = \frac{1}{2} \int_0^{z_m} dz V(Q,z) \sum_\tau c_\tau^{RN} (F_{\tau,0}^L(z) F_{\tau,1}^L(z) + F_{\tau,0}^R(z) F_{\tau,1}^R(z)) \tag{19}$$

$$G_2(Q^2) = \frac{1}{2} \int_0^{z_m} dz V(Q,z) \sum_\tau c_\tau^{RN} (F_{\tau,0}^R(z) F_{\tau,1}^R(z) - F_{\tau,0}^L(z) F_{\tau,1}^L(z)) \tag{20}$$

$$G_3(Q^2) = \frac{1}{2} \int_0^{z_m} dz \partial_z V(Q,z) \sum_\tau c_\tau^{RN} (F_{\tau,0}^L(z) F_{\tau,1}^L(z) - F_{\tau,0}^R(z) F_{\tau,1}^R(z)) \tag{21}$$

$$G_4(Q^2) = \frac{M}{2} \int_0^{z_m} dz V(Q,z) \sum_\tau c_\tau^{RN} (F_{\tau,0}^L(z) F_{\tau,1}^R(z) + F_{\tau,1}^L(z) F_{\tau,0}^R(z)) \tag{22}$$

Pauli and Dirac form factors of the $N+\gamma^* \to R(1710)$ can be expressed as following using (19-22) integral formulas:

$$F_1(Q^2) = G_1(Q^2) + g_V G_2(Q^2) + \eta_V G_3(Q^2), \tag{23}$$

$$F_2(Q^2) = \eta_V G_4(Q^2). \tag{24}$$

From time to time the functions $F_1(Q^2)$ and $F_2(Q^2)$ are called respectively, the charge and moment form factors of the nucleon. The helicity amplitudes, $A_{1/2}$ and $S_{1/2}$ can be defined within the form factors $F_1(Q^2)$ and $F_2(Q^2)$ [14]:

$$A_{1/2}(Q^2) = R\big(F_1(Q^2) + F_2(Q^2)\big), \tag{25}$$

$$S_{1/2} = \frac{R}{\sqrt{2}} |q| \frac{M_R+M}{Q^2} \{ F_1(Q^2) - \tau F_2(Q^2)\}, \tag{26}$$

where

$$\tau = \frac{Q^2}{(M_R+M)^2},$$

and

$$R = \frac{e}{2}\sqrt{\frac{Q^2}{M_R M K}}.$$

4 Numerical results

We shall present the numerical calculations for the $N+\gamma^* \to R(1710)$ transition electromagnetic form factors according to the (25) and (26) formulas.

The parameters and constants in these formulas are fixed as $c_3^{RN} = 0.72, \eta_P = 0.453$ [2] and $z_m = \frac{1}{\Lambda_{QCD}} = 0.205\ GeV$.

The form factors results for the $N+\gamma^* \to R(1710)$ transition are presented in Figs. 1, 2 and compared to the data from CLAS [15], MAID [16] experimental data, the nonrelativistic quark model [17]. Our results for the Pauli and Dirac form factors are close to these experimental data.

The hard-wall results for the $A_{1/2}(Q^2)$ and $S_{1/2}(Q^2)$ helicity amplitudes for this transition are given in Figs. 3, 4. The graphs of these amplitudes are close to the experimental data at some points. It is interesting to see in the figure that, the hard wall model describes very well the CLAS and MAID data Dirac form factor $F_1(Q^2)$ for $Q^2 < 1\ GeV^2$.

In general, we can see from the figures that our results are close experimental data in the $0 \le Q^2 \le 1$ interval.

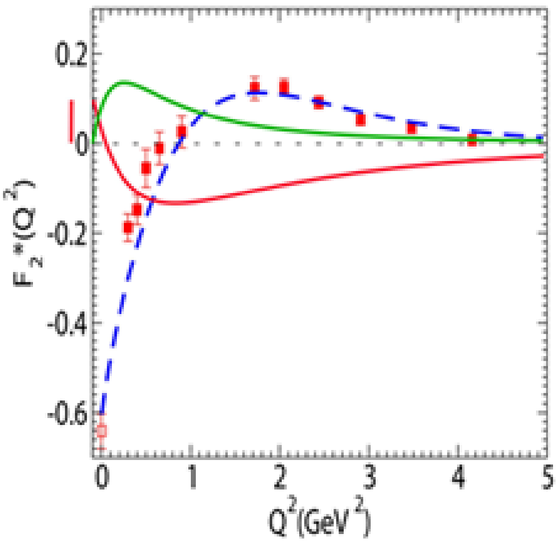

Figure 1: $N+\gamma^{*}\rightarrow R(1710)$ transition Dirac form factor (red lines) compared with CLAS experimental data (squares with error bars) [15], MAID fit (dashed lines) [16] and nonrelativistic quark model (green lines) [17]. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

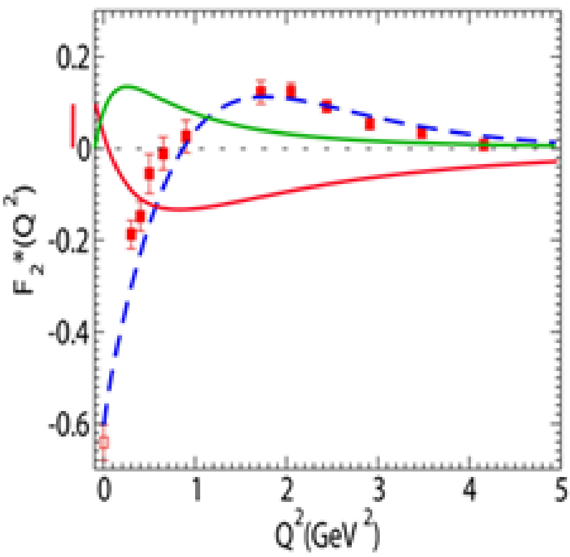

Figure 2: $N+\gamma^{*}\rightarrow R(1710)$ transition Pauli form factor (red lines) compared with CLAS experimental data (squares with error bars) [15], MAID fit (dashed lines) [16] and nonrelativistic quark model (green lines) [17]. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

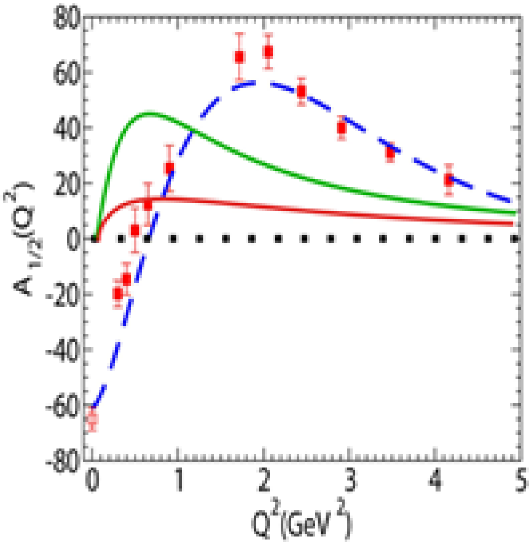

Figure 3: $N+\gamma^*\rightarrow R(1710)$ $A_{1/2}(Q^2)$ helicity amplitude in units of 10^{-1} $GeV^{-1/2}$ (red lines) is compared with CLAS experimental data (squares with error bars) [15], MAID fit (dashed lines) [16] and nonrelativistic quark model (green lines) [17]. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

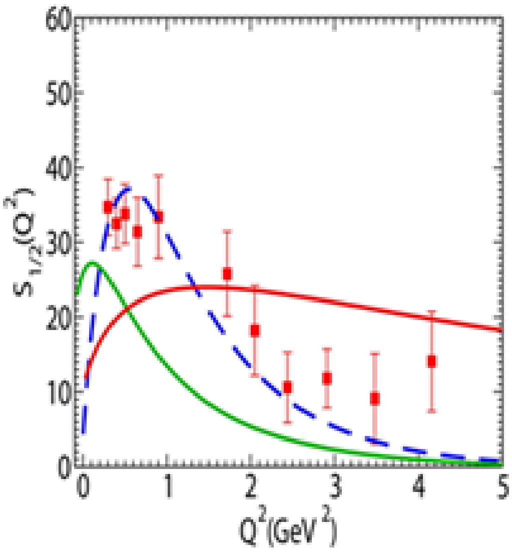

Figure 4: $N+\gamma^*\rightarrow R(1710)$ $S_{1/2}(Q^2)$ helicity amplitudes in units of 10^{-1} $GeV^{-1/2}$ (red lines) is compared with CLAS experimental data (squares with error bars) [15], MAID fit (dashed lines) [16] and nonrelativistic quark model (green lines) [17]. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

5 Conclusion

In this work, we have used the hard-wall AdS/QCD model, to predict the transition form factors and the helicity amplitudes for the $N+\gamma^* \to R(1710)$ reaction. The hard-wall results are close to experimental data and nonrelativistic quark model at some points. Therefore, the future measurements of the helicity amplitudes in the large Q^2 region can be used to test the assumption that the $N(1710)$ state is the second radial excitation of the nucleon and this model can be used to study the radial excited states in the Δ sector (isospin 3/2), and also in the strange baryon sector, as well as.

Authors' Contributions

All authors have the same contribution.

Data Availability

The manuscript has no associated data or the data will not be deposited.

Conflicts of Interest

The authors declare that there is no conflict of interest.

Ethical considerations

The authors have diligently addressed ethical concerns, such as informed consent, plagiarism, data fabrication, misconduct, falsification, double publication, redundancy, submission, and other related matters.

Funding
This research did not receive any grant from funding agencies in the public, commercial, or non-profit sectors.

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